Micro Data for Macro Models

Topic 4: Firm Lifecycle

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Stylized Facts About Firm Dynamics

1. New entrants smaller than the average firm

2. Young firms more likely to exit than average firm

3. Conditional on survival:
   - Young firms grow faster than average
   - Small firms grow faster than average, conditional on age

4. Distribution of firm size has fat tail

Benchmark Model of Firm (Non) Dynamics

- Consider firms i with production function

$$y_{it} = e^{z_{it}} k_{it}^{\theta} n_{it}^{\nu}, \text{ where } \theta + \nu < 1$$

- Productivity shocks feature mean reversion

$$z_{it+1} = \rho z_{it} + \varepsilon_{it+1}$$

- Suppose firms enter with average productivity z_{i0} and low capital $k_{i0} < \mathbb{E}[k_{it}]$

- Investment satisfies $1 = \mathbb{E}[MPK_{it+1} + (1-\delta)]$

$\implies$ **All growth occurs in first period**

Mechanisms Generating Firm Dynamics

1. Selection upon entry and exit (Hopenhayn 1992)
    - Surviving firms have higher productivity than entrants
    - Mean reversion + selection $\Longrightarrow$ growth patterns
2. Capital adjustment costs (Clementi and Palazzo 2015)
    - Keeps investment from all happening in first period
3. Financial frictions (Ottonello and Winberry 2017)
    - Costly to finance investment
4. Demand accumulation (Foster, Haltiwanger, and Syverson 2016)
    - Another form of investment

Clementi and Palazzo (2015)

- Adds capital adjustment costs to Hopenhayn (1992) model

- Model matches key stylized facts about firm dynamics
  1. New entrants smaller than average
  2. Young firms more likely to exit
  3. Conditional on survival, young/small firms grow faster

- Generates persistence in response to aggregate shocks

Model Overview

Incumbent Firms

- Idiosyncratic + aggregate productivity shocks
- Fixed operating cost + exit
- Fixed + quadratic adjustment costs

Potential Entrants

- Fixed mass
- Signal of productivity if enter
- Fixed entry cost

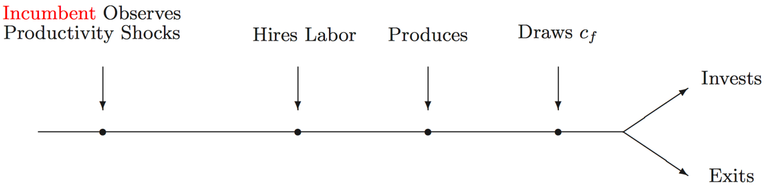
Partial equilibrium

- Fixed discount factor $1/R$
- Labor supply function $L(w) = w^{\gamma}$

Incumbent Firms

- Production function $y_{it} = e^{Z_t} e^{z_{it}} k_{it}^{\theta} n_{it}^{\nu}$
    - Aggregate shock $Z_{t+1} = \rho_Z Z_t + \varepsilon_{t+1}^{Z}$
    - Idiosyncratic shock $z_{it+1} = \rho_z z_{it} + \varepsilon_{it+1}^{z}$
- Capital accumulation follows $k_{it+1} = (1-\delta)k_{it} + i_{it}$
    - Fixed cost $c_0 k_{it}$ if $i_{it} \neq 0$
    - Quadratic adjustment cost $-\frac{c_1}{2}\left(\frac{i_{it}}{k_{it}}\right)^2 k_{it}$
- To continue into next period, must pay fixed operating cost $c_f \sim \log N(\mu_{c_f}, \sigma_{c_f})$

Timing and Decision Problem



$$v^1(z,k;\mathbf{s}) = \max_n e^Z e^z k^\theta n^\nu - w(\mathbf{s})n + \mathbb{E}_{c_f}\left[\max\{v^0(k), v^2(z,k;\mathbf{s}) - c_f\}\right]$$

$$v^2(z,k;\mathbf{s}) = \max_{k'} -(k' - (1-\delta)k) - \mathrm{AC}(k',k) + \frac{1}{R}\mathbb{E}_{z',Z'|z,Z}\left[v^1(z',k';\mathbf{s}')\right]$$

Potential Entrants

- Fixed mass M of potential entrants
- Draw signal of future productivity $q \sim \text{Pareto}(q)$
  - $z' = \rho_S q + \eta', \eta' \sim N(0, \sigma_S)$
- To become an incumbent, pay fixed entry cost c_e

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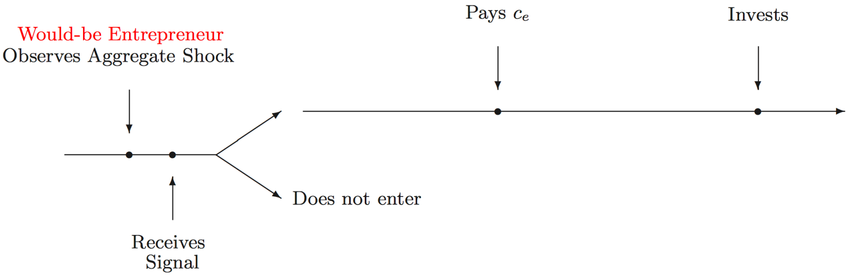

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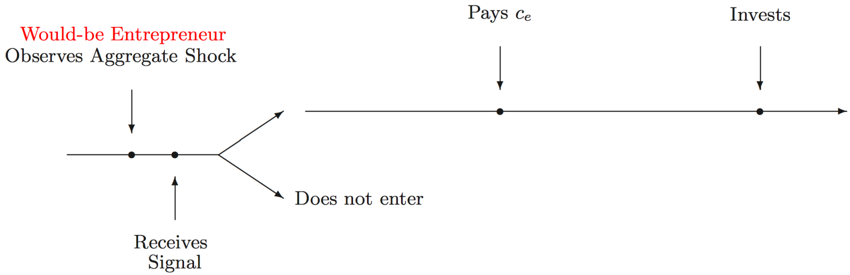


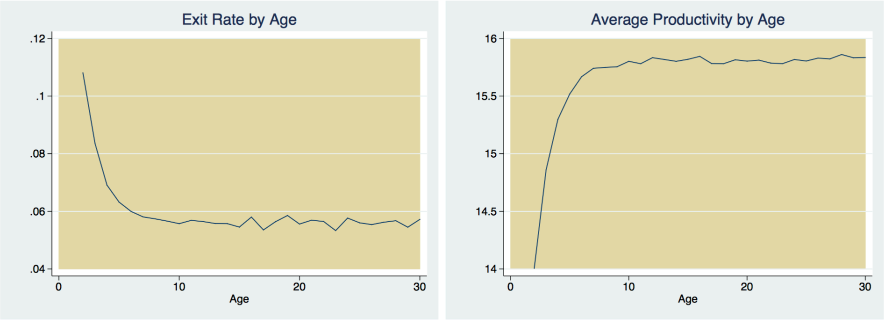
$$v(q; \mathbf{s}) = -k' + \frac{1}{R}\mathbb{E}_{z', Z' | z, q}\left[v^1(z', k'; \mathbf{s}')\right]$$

Calibration

Description	Symbol	Value
Capital share	α	0.3
Span of control	θ	0.8
Depreciation rate	δ	0.1
Interest rate	R	1.04
Labor supply elasticity	γ	2.0
Mass of potential entrants	M	1,766.29
Persistence idiosync. shock	ρ_s	0.55
Variance idiosync. shock	σ_s	0.22
Operating cost – mean parameter	μ_{c_f}	-5.63872
Operating cost – var parameter	σ_{c_f}	0.90277
Fixed cost of investment	c_0	0.00011
Variable cost of investment	c_1	0.03141
Pareto exponent	ξ	2.69
Entry cost	c_e	0.005347

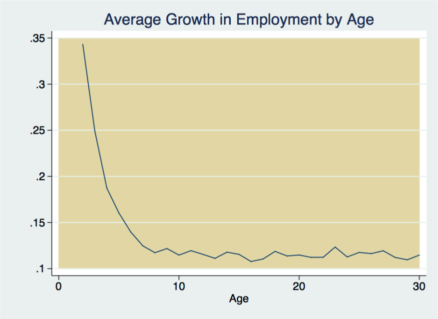
Statistic	Model	Data
Mean investment rate	0.153	0.122
Std. Dev. investment rate	0.325	0.337
Investment autocorrelation	0.059	0.058
Inaction rate	0.067	0.081
Entry rate	0.062	0.062
Entrants' relative size	0.58	0.60
Exiters' relative size	0.47	0.49

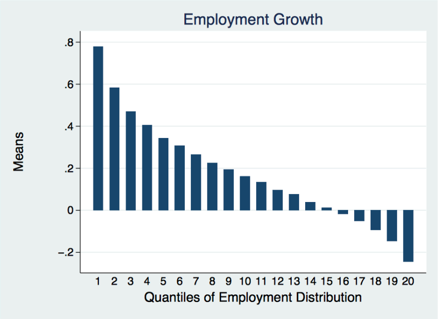
Survival Probability by Age



- Survival probability decreases with age
    - Productivity increases with age

Growth by Age and Size





- Growth rate decreasing in size
  - Mean reversion in productivity
- Growth rate decreasing in age
  - Young firms have lower capital

Calibrating Aggregate TFP Shocks

Parameters

Description	Symbol	Value
Labor supply elasticity	γ	2.0
Persist. aggregate shock	ρ_z	0.685
Std. Dev. aggregate shock	σ_z	0.0163

Targets

Statistic	Model	Data
Standard deviation output growth	0.032	0.032
Autocorrelation output growth	0.069	0.063
Std. dev. employment growth (rel. to output growth)	0.656	0.667

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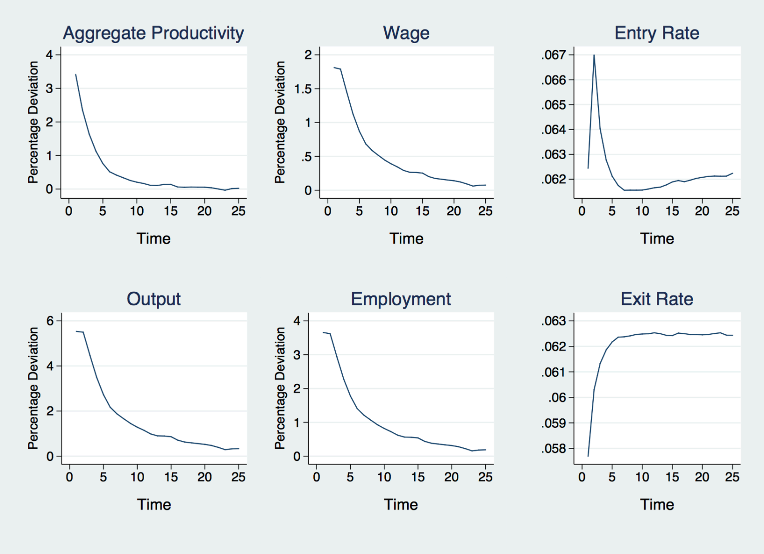
Entry and Exit Over the Cycle

Entry Rate	Exit Rate	Entrants' Size	Exiters' Size
0.402	-0.779	-0.725	-0.892

Aggregate Impulse Respones



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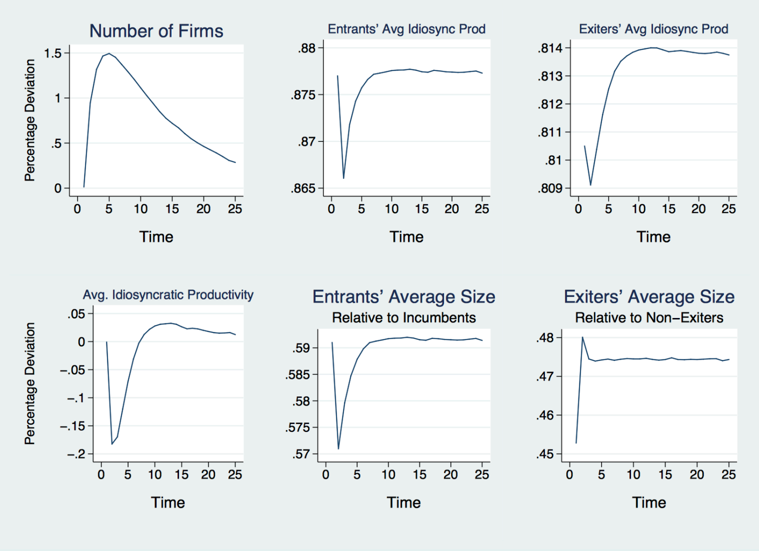
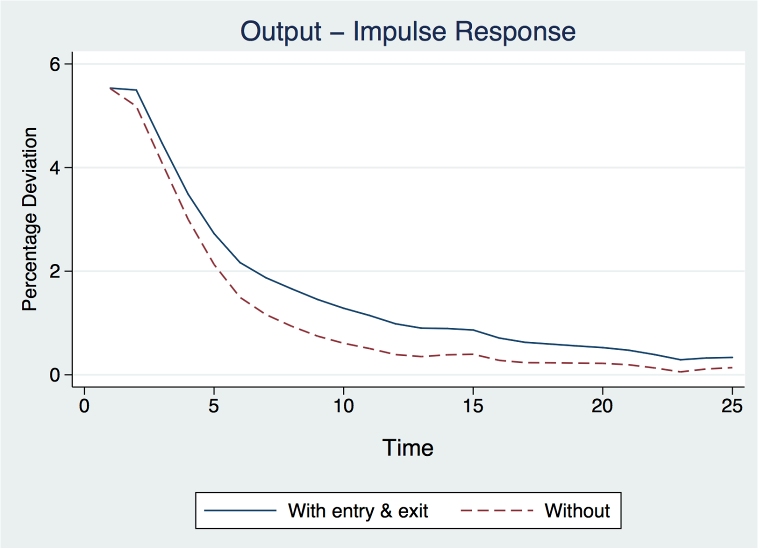


Propagation



Three New Propagation Mechanisms in This Model

$Y_t = A_t K_t^{1-\alpha} L_t^{\alpha}$, where

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2. **Internal propagation**:
   - Capital accumulation

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   - Capital accumulation
   - Firm accumulation
   - Selection

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2. **Internal propagation**:
   - Capital accumulation
   - Firm accumulation
   - Selection
   - Allocation