ECO 503/517 Computational Precept Fall 2013
Matlab Probem Set

This problem set asks you to exercise many of the tools you learned this morning in Matlab. These tools will be crucial for solving problem sets throughout the first year, so be sure that you can work through all of these problems. Please don't hesitate to contact me if you have trouble with anything here so we can go through it.

This problem set has you solve and analyze the efficient allocation of a static production economy. There is a single household in this economy with utility over consumption c and hours worked h given by the function $u(c,h) = \frac{c^{1-\sigma}-1}{1-\sigma} - \psi\frac{h^{1+\eta}}{1+\eta}$ for parameters $\sigma = 2$, $\psi = 2.4$, and $\eta = \frac{1}{2}$. There is a single firm with production technology given by $f(h) = Ah^\alpha$ for parameters $A = 1$ and $\alpha = \frac{2}{3}$. The social planner maximizes the household's utility subject to the constraints $c = Ah^\alpha$ and $0 \leq h \leq 1$; that is, the social planner performs

$$\max_h \frac{(Ah^\alpha)^{1-\sigma}-1}{1-\sigma} - \psi\frac{h^{1+\eta}}{1+\eta} \text{ such that } 0 \leq h \leq 1$$

In what follows assume the solution to this problem is always interior.

1. Early in the development of computational methods problems were often solved by discretization, that is, by forcing the choice variables (here, h) to take on a discrete number of points and picking the value that yields the highest value of the objective function.

   (a) Discretize the choice space by creating an $N \times 1$ vector $H = [h_1 = .01, h_2, ..., h_N = .99]$ for $N = 200$.

   (b) Using the vector H, create a vector U whose i^{th} element is equal to $\frac{(Ah_i^\alpha)^{1-\sigma}-1}{1-\sigma} - \psi\frac{h_i^{1+\eta}}{1+\eta}$. Use a for loop. Time how long it takes your code to construct U using the internal timing functions. (Hint: in Matlab, use "tic" and "toc." Use the help files or online resources to find the correct syntax.)

   (c) Using the vector H, create a vector U whose i^{th} element is equal to $\frac{(Ah_i^\alpha)^{1-\sigma}-1}{1-\sigma} - \psi\frac{h_i^{1+\eta}}{1+\eta}$. Use matrix operations and time how long it takes your code to construct U. Which method, (a) or (b), is faster? By how much? Which is easier to code and read?

   (d) Find the value of $h \in H$ (equivalently, the index $i \in \{1, ..., N\}$) which yields the highest value of U. (Hint: in Matlab, use the "max" command. Use the help files or online resources to find the correct syntax.)

2. The discretization routine above was quite brutal and in harder problems prohibitively inefficient. Therefore we will now try a method which exploits the continuity and concavity of the objective function and convexity of the constraint set.

   (a) Write a function $u(h)$ which takes as input a scalar h and returns as output $\frac{(Ah^\alpha)^{1-\sigma}-1}{1-\sigma} - \psi\frac{h^{1+\eta}}{1+\eta}$.

(b) Maximize this function. (Hint: in Matlab, use the "fminunc" command on the negative of the function. Again, use the help files or online resources to find the correct syntax.)

3. Now that we have a relatively efficient way to solve for the static allocation, we'll add some random shocks and interpret a series of static allocation as a time series. In particular, assume that there are T periods and index $t \in \{1, ..., T\}$. Each period, aggregate productivity A_t is a realization of a lognormally distributed random variable: $\log A_t \sim N(0, \sigma_A^2)$ iid over t for $\sigma_A = .02$. Conditional on A_t, the social planner chooses h_t to solve for the static allocation as above. There are no links between time periods.

   (a) Create a $T \times 1$ vector A_t which contains the realizations of the lognormal distributed random variable given above. Use $T = 50$.

   (b) Write a function $allocation(A)$ which solves that static allocation problem. That is, given A, the function returns a value of h. Use your answer from 2b.

   (c) Create a $T \times 1$ vector H_t whose t^{th} entry is the allocation h_t given the value of A_t.

   (d) Plot your results in two graphs: one of productivity A_t over time and another of hours worked h_t over time.

4. Now suppose you are an econometrician who wants to estimate a labor supply function for this economy. You are given a time series $\{A_t, h_t\}_{t=1}^T$ and postulate the model $\log h_t = \alpha + \beta \log A_t + \varepsilon_t$ where $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$ iid over t. Note that this is not the true model.

   (a) Create a time series $\{A_t, h_t\}_{t=1}^T$ for $T = 1000$ using your answer to question 3.

   (b) Estimate the econometric model by OLS. First, create an $N \times 1$ vector Y whose t^{th} entry is $\log h_t$. Then create an $N \times 2$ vector X where the first column is a column of 1's (for the constant) and the second column contains the entries of $\log A_t$. (Hint: in Matlab, use "concatenate"). The OLS coefficients $\widehat{\alpha}$ and $\widehat{\beta}$ are given by the formula $(X'X)^{-1}X'Y$.

   (c) Create a $T \times 1$ vector of predicted values of the form $\log \widehat{h_t} = \widehat{\alpha} + \widehat{\beta} \log A_t$. In general $\log \widehat{h_t} \neq \log h_t$. How do the two compare? To answer this, compute three statistics. First, compute the mean difference between the two, that is, $mean(\log \widehat{h_t} - \log h_t)$. This should be close to 0 because there is a constant in the regression. Second, compute the maximum absolute difference between the two, that is $\max_t |\log \widehat{h_t} - \log h_t|$. Finally, compute the linear correlation coefficient between $\log \widehat{h_t}$ and $\log h_t$. The squared value of this statistic is called the R^2 and captures the fraction of the variation in $\log h_t$ "explained" by $\log \widehat{h_t}$.